

1. **The Pendulum Problem:** If a pendulum is pulled back and released, it will swing back and forth over time with a noticeable pattern before it comes to rest. If the pendulum swings over a relatively small time interval its motion can be modeled by a sinusoidal function.

Suppose a pendulum is swinging in front of a motion detector, which measures and calculates the distance from the pendulum to the motion detector over a time interval.

After 1.2 seconds, the pendulum is 0.82 m, the maximum distance from the motion detector, and at 2.8 seconds the pendulum is at a distance of 0.36 m, the minimum distance from the motion detector.

- a. Sketch a graph of two complete periods of the sinusoidal function that models the distance of the pendulum to the motion detector. Label all important values on the two axes.



- b. State the amplitude, period, phase shift, and vertical shift of the function drawn above.

Amplitude: \_\_\_\_\_ Period: \_\_\_\_\_ Phase Shift: \_\_\_\_\_ Vertical Shift: \_\_\_\_\_

- c. Write the equation of the sinusoidal function  $y(t)$  that models the distance  $y$  of the pendulum with respect to time  $t$ .

$y(t) =$  \_\_\_\_\_

- d. According to the equation, how far was the pendulum from the motion detector when we began collecting data?

$t =$  \_\_\_\_\_

**2. Hours of Sunlight Problem:** The number of hours of sunlight in a day for a given location can be modeled by a sinusoidal function. The longest day of the year (in terms of hours of sunlight) occurs on the day of the summer solstice. The summer solstice is the time when the sun is the farthest north. In 2016 in Tucson, the summer solstice will occur on June 21 (the 172nd day of the year). The shortest day of the year occurs on the day of the winter solstice. The winter solstice is the time when the sun is farthest south. This year in Tucson, the winter solstice will occur on December 21 (the 355th day of the year). The number of hours of sunlight in Tucson on the summer solstice will be 14 hrs, 16 min (14.27 hours) and the number of hours of sunlight on the winter solstice will be 10 hrs, 2 min (10.03 hours).

- a. Draw a graph of the number of hours of sunlight with respect to the days of the year for the year 2016, which is a leap year!. Label the important values on the two axes.



- b. State the amplitude, period, phase shift, and vertical shift of the function drawn above.

Amplitude: \_\_\_\_\_ Period: \_\_\_\_\_ Phase Shift: \_\_\_\_\_ Vertical Shift: \_\_\_\_\_

- c. Write the equation of the sinusoidal function  $y(t)$  that models the number of hours of sunlight in Tucson  $y$  with respect to the number of days of the year  $t$ , since the new year.

$y(t) =$  \_\_\_\_\_

- d. According to the equation, how many hours of sunlight will there be in Tucson today, \_\_\_\_\_, which is the \_\_\_\_\_ day of the year?

\_\_\_\_\_ hrs, or \_\_\_\_\_ hrs and \_\_\_\_\_ min

3. **The Tide Problem:** Suppose you are on the beach in San Diego, California. At 6:00 am, at low tide, you read that the depth of the water at the end of a pier will be 1.93 ft. At 12:45 pm, at high tide, the depth of the water will be 6.27 ft. Assume that the depth of the water varies sinusoidally with respect to time.

a. Approximately at what time will the next low tide occur? \_\_\_\_\_ (am or pm) which is \_\_\_\_\_ hrs after midnight.

b. Draw a graph of the depth of the tide as a function of the number of hours that have elapsed since 12:00 midnight at the beginning of September 20. (In other words, 6:00 pm would be at  $t = 18$ .) Label the important values on the two axes.

(Note: 12:45 pm is how many hours after midnight? \_\_\_\_\_)



c. State the amplitude, period, phase shift, and vertical shift of the function drawn above.

Amplitude: \_\_\_\_\_ Period: \_\_\_\_\_ Phase Shift: \_\_\_\_\_ Vertical Shift: \_\_\_\_\_  
(Hint: Not 24!)

d. Write the equation of the sinusoidal function  $y(t)$  that models the depth  $y$  of the tide with respect to the number of hours  $t$  since midnight at the beginning of September 20.

$y(t) =$  \_\_\_\_\_

e. According to the equation, at what time does the first low tide occur on September 21?

It occurs at \_\_\_\_\_ hrs , \_\_\_\_\_ min

f. Use your graph to determine at what time(s) on September 21 that the water will be 4.5 ft deep. (Hint: There are three of them!)

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